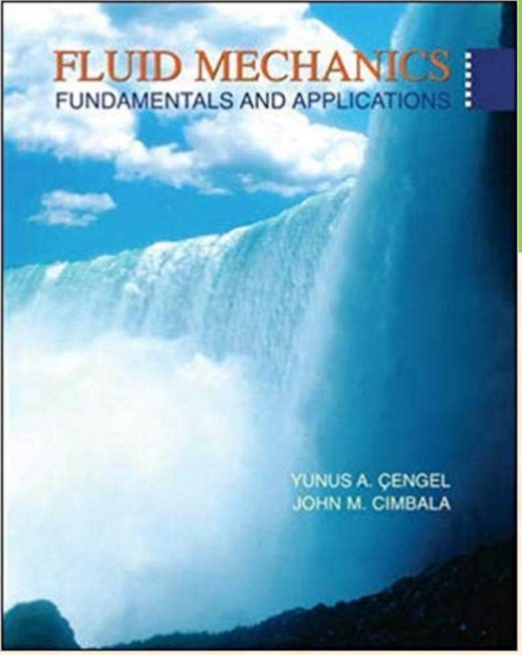

YUNUS A. ÇENGEL and JOHN M. CIMBALA, "Fluid Mechanics: Fundamentals and Applications",1st ed., McGraw-Hill,2006.

Course name

Incompressible Fluid Mechanics

Lecture-03- Chapter-03

Fluid flow concept and Basic equations

Lecture slides by

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University of Tikrit

Outline

- ***Momentum Equation***
- ***Fixing and moving vanes***
- ***The moment theory for propeller***
- ***Examples***
- ***Homeworks***
- ***Degrees Distribution***

3.9 Momentum Equation

The **momentum equation** is commonly used to calculate the forces (usually on support systems or connectors) induced by the flow.

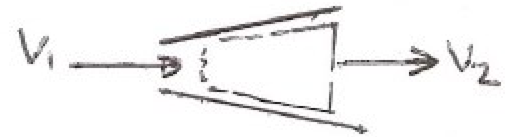
THE LINEAR MOMENTUM EQUATION →

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d}{dt}(m\vec{V})$$

Approximate RTT for well-defined inlets and outlets:

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b \, dV + \sum_{out} \underbrace{\rho_{avg} b_{avg} V_{r,\,avg} A}_{\text{for each outlet}} - \sum_{in} \underbrace{\rho_{avg} b_{avg} V_{r,avg} A}_{\text{for each inlet}} \quad \text{..... (3.8)}$$

Here $b = \vec{V}$, equ (3.8) becomes



$$\frac{D(m\vec{V})_{sys}}{Dt} = \frac{\partial (m\vec{V})}{\partial t} + \rho_2 V_2 \vec{V}_2 A_2 - \rho_1 V_1 \vec{V}_1 A_1$$

Newton 2nd law — μ=const.

steady $\frac{\partial}{\partial t} = 0$

$$\sum \vec{F} = \rho_2 V_2 A_2 \vec{V}_2 - \rho_1 V_1 A_1 \vec{V}_1$$

for X-direction

$$\sum F_x = \rho_2 V_2 A_2 V_{2x} - \rho_1 V_1 A_1 V_x \quad \because \dot{m} = \dot{m}_1 = \dot{m}_2$$

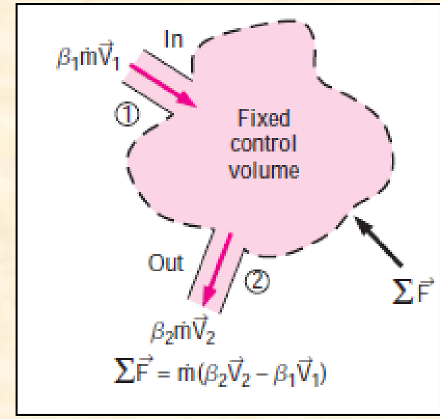
$$\boxed{\sum F_x = \dot{m}(V_{x_2} - V_{x_1})} \quad \text{——— (3.31)}$$

3.9 Momentum Equation

Momentum-flux correction factor: $\beta = \frac{1}{A_c}\int_{A_c}\left(\frac{V}{V_{avg}}\right)^2 dA_c$

The momentum equation for steady one-dimensional flow is

$$\sum \vec{F} = \sum_{out} \beta \dot{m}\vec{V} - \sum_{in} \beta \dot{m}\vec{V}$$



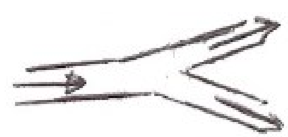
✓ **we would find that β ranges from about 1.01 to 1.04.**

* For one inlet and outlet

$\Sigma F = \dot{m}(\vec{V}_{out} - \vec{V}_{in})$ ---- 3-32

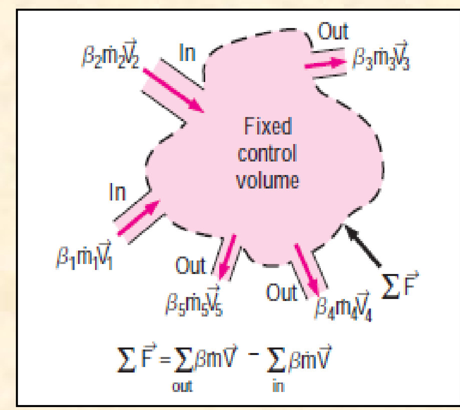
* For more than one inlet one outlet

$\Sigma F = \Sigma(\dot{m}v)_{out} - \Sigma(\dot{m}\vec{V})_{in}$ --- 3-33



↓

$F = F_1 + F_2 + F_3$



$$\sum \vec{F} = (\beta_3\dot{m}_3\vec{V}_3 + \beta_4\dot{m}_4\vec{V}_4 + \beta_5\dot{m}_5\vec{V}_5) - (\beta_1\dot{m}_1\vec{V}_1 + \beta_2\dot{m}_2\vec{V}_2)$$

3.9 Momentum Equation

The momentum equation for steady one-dimensional flow is

$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

IF $V_2 = V_1 = V$

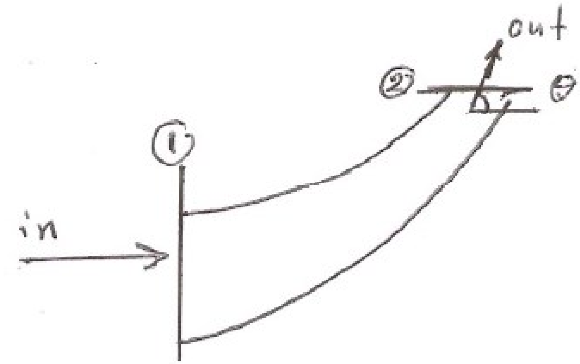
① F_1 = Change of momentum due to change in direction in X- direction

$F_{x_1} = \dot{m}\,(V_2 \cos\theta_2 - V_1 \cos\theta_1)$

if $\theta_1 = 0$, $\theta_2 = 60°$

$F_{x_1} = \dot{m} V\,(\cos 60 - \cos 0)$

$F_{x_1} = -0.5\,\dot{m} V$



② F_2 = change in momentum due to change in pressure

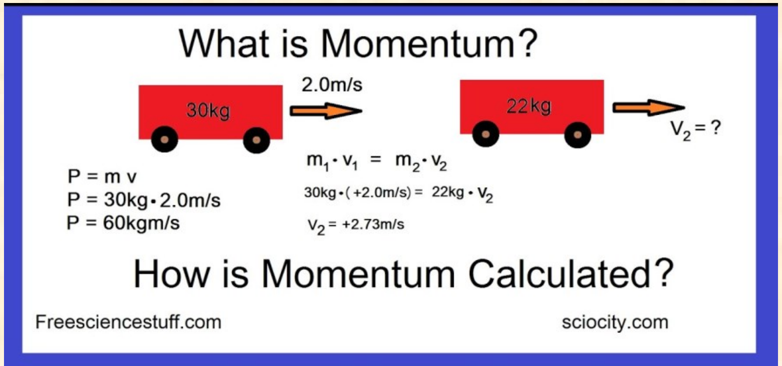
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

force $= \sum \overrightarrow{PA}$

ⓐ $\dfrac{P_1 - P_2}{\gamma}$ = $\text{losses}_{1\rightarrow 2}$ if $V_1 = V_2$ (constant area)

ⓑ $\dfrac{P_1 - P_2}{\gamma} = \dfrac{V_2^2 - V_1^2}{2g} + \text{losses}_{1-2}$ if $V_1 \neq V_2$ (change area)

https://www.google.com.tr/search?q=momentum+equation+physics&source=lnms&tbm=isch&sa=X&ved=2ahUKEwjxspLey6noAhXomHIEHZW8Dt0Q_AUoAXoECA4QAw&biw=1366&bih=667#imgrc=ewpRu_Dbpnt9EM



3.9 Momentum Equation

③ F_3 = Change in momentum due to gravity force (W↓)

General

$$\sum \vec{F} = \sum_{out} \beta \dot{m}\vec{V} - \sum_{in} \beta \dot{m}\vec{V}$$

In X-direction

$$\sum (PA)_{in} \cos\theta - \sum (PA)_{out} \cos\theta - F_x = \sum (\dot{m}V)_{out} \cos\theta - \sum (\dot{m}V)_{in} \cos\theta \quad \text{———} (3\text{-}34)$$

in y-direction

$$\sum (PA)_{in} \sin\theta - \sum (PA)_{out} \sin\theta + W - F_y = \sum (\dot{m}V)_{out} \sin\theta - \sum (\dot{m}V)_{in} \sin\theta \quad \text{----} (3\text{-}35)$$

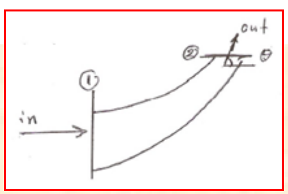
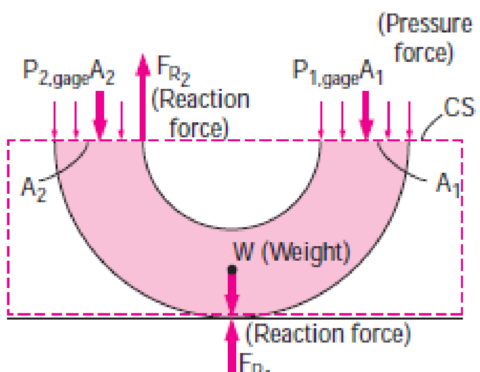
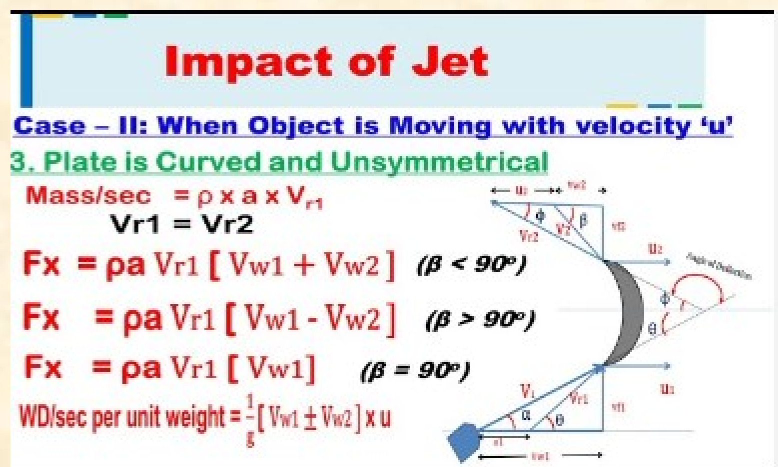


FIGURE 6–12
In most flow systems, the force $\vec{F}$ consists of weights, pressure forces, and reaction forces. Gage pressures are used here since atmospheric pressure cancels out on all sides of the control surface.

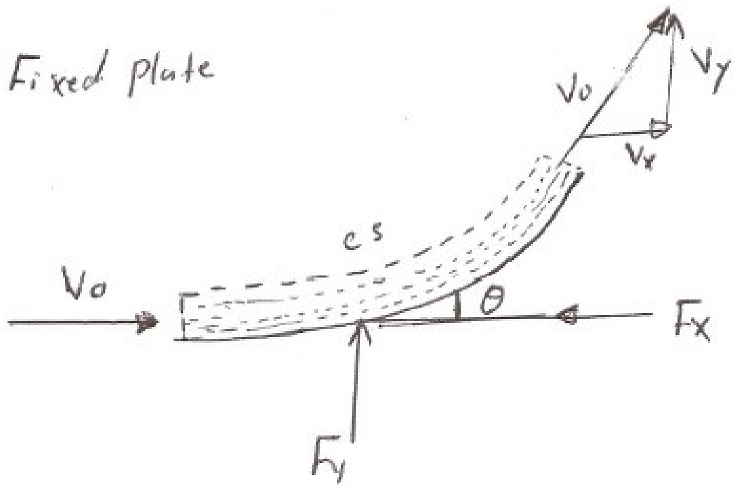


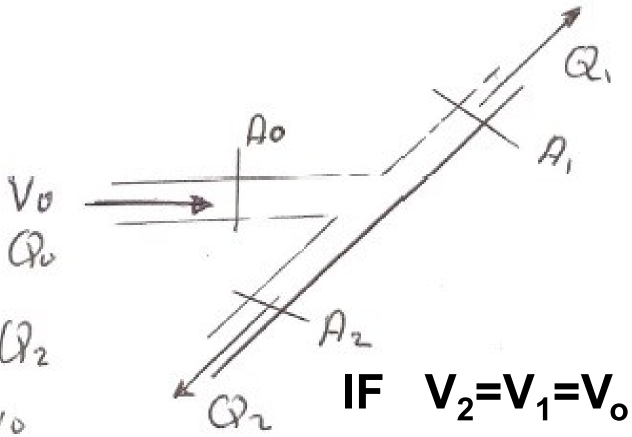
An 180° elbow supported by the ground



3.10 Fixing and moving vanes

(a) Fixed plate





$Q_0 = Q_1 + Q_2$

$Q_0 = A_0 V_0$

$Q_1 = A_1 V_0$

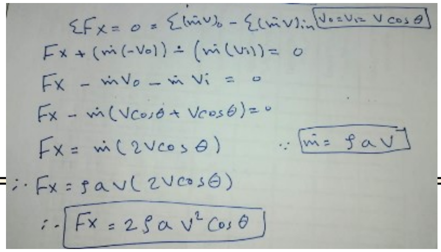
$Q_2 = A_2 V_0$

IF $V_2 = V_1 = V_o$

Force exerted by the jet on a stationary plate (Symmetrical Plate)

Plate is Curved and Jet strikes at tip

$$F_x = 2\rho a V^2 \cos\theta$$



$\Sigma F_x = 0 = \Sigma(\dot{m}V)_{out} - \Sigma(\dot{m}V)_{in}$ $\boxed{V_0 = V_1 = V\cos\theta}$

$F_x + (\dot{m}(-V_0)) - (\dot{m}(V_1)) = 0$

$F_x - \dot{m}V_0 - \dot{m}V_1 = 0$

$F_x - \dot{m}(V\cos\theta + V\cos\theta) = 0$

$F_x = \dot{m}(2V\cos\theta)$ $\quad \because \boxed{\dot{m} = \rho a V}$

$\therefore F_x = \rho a V(2V\cos\theta)$

$\therefore \boxed{F_x = 2\rho a V^2 \cos\theta}$

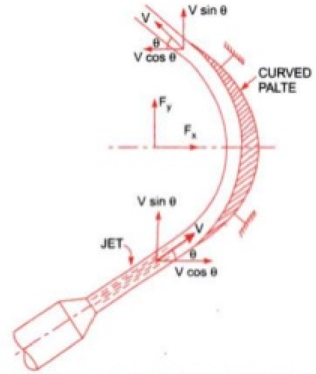


Fig. 17.4 Jet striking curved fixed plate at one end.

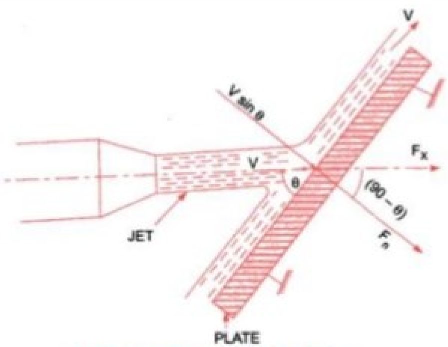
Force exerted by the jet on a stationary plate

Plate is inclined to the jet

$$F_N = \rho a V^2 \sin\theta$$

$$F_x = F_N \sin\theta$$

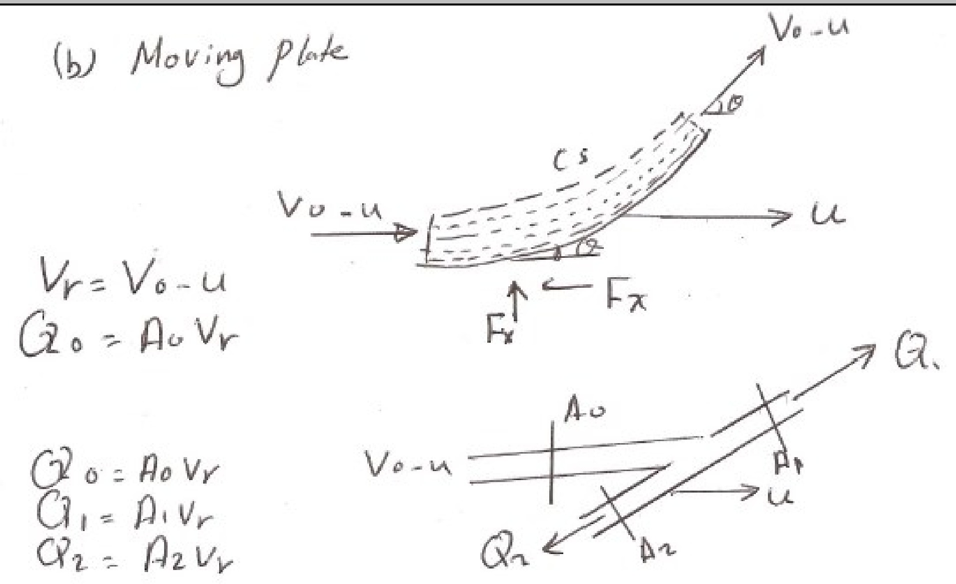
$$F_y = F_N \cos\theta$$

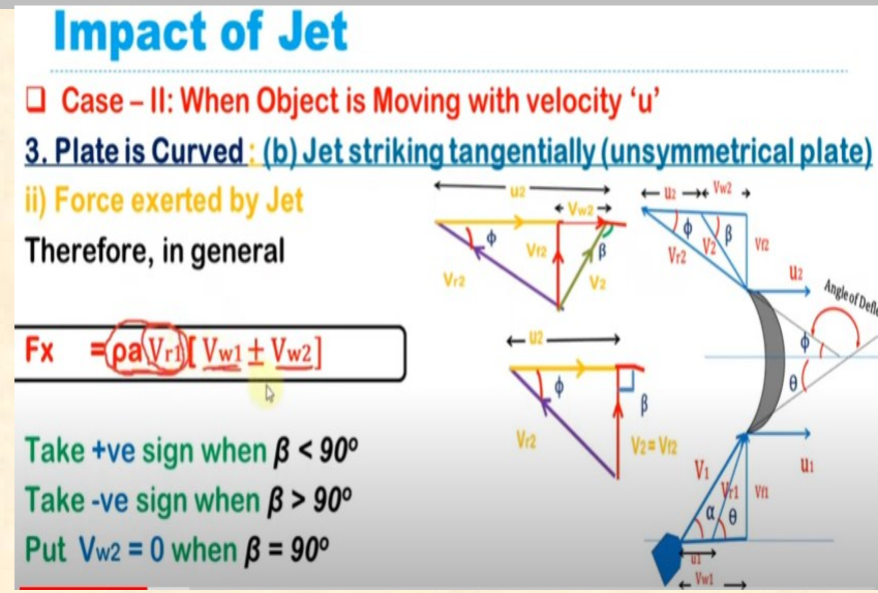


Jet striking stationary inclined plate.

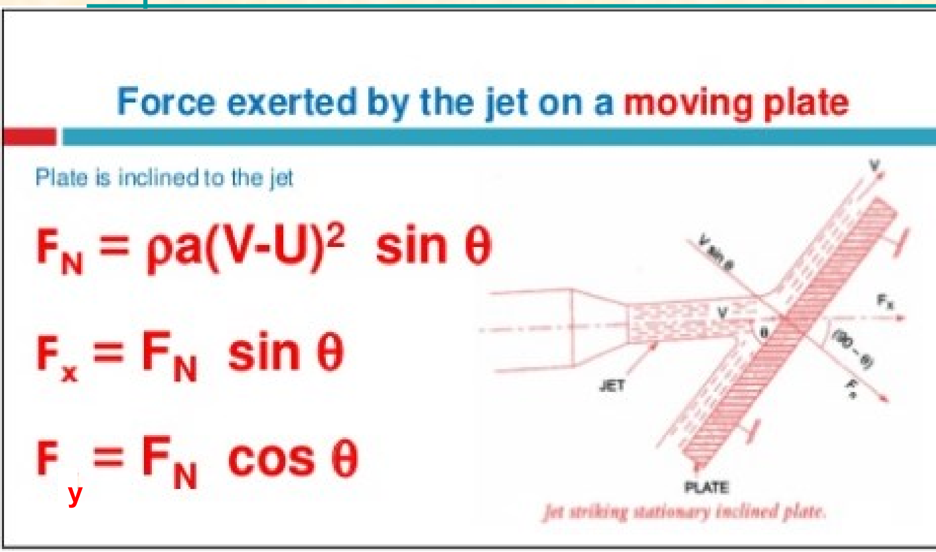
- https://www.google.com.tr/search?q=fixed+vane+plate&tbm=isch&ved=2ahUKEwj0u4_O16roAhUMMt8KHQ0VC5QQ2-cCegQIABAA&oq=fixed+vane+plate&gs_l=img.3...52101.56978..59059...0.0..0.230.1449.0j3j4......0....1..gws-wiz-img.GJpOMrfgV1g&ei=6pF1XvSeLIzk_AaNqqygCQ&bih=667&biw=1366&hl=en#imgrc=vfEgDmc3wls5kM&imgdii=T-rxd9TEBO5vtM
- https://www.google.com.tr/search?q=fixed+vane+plate&tbm=isch&ved=2ahUKEwj0u4_O16roAhUMMt8KHQ0VC5QQ2-cCegQIABAA&oq=fixed+vane+plate&gs_l=img.3...52101.56978..59059...0.0..0.230.1449.0j3j4......0....1..gws-wiz-img.GJpOMrfgV1g&ei=6pF1XvSeLIzk_AaNqqygCQ&bih=667&biw=1366&hl=en#imgrc=T-rxd9TEBO5vtM&imgdii=AURwy5zL5w0LSM

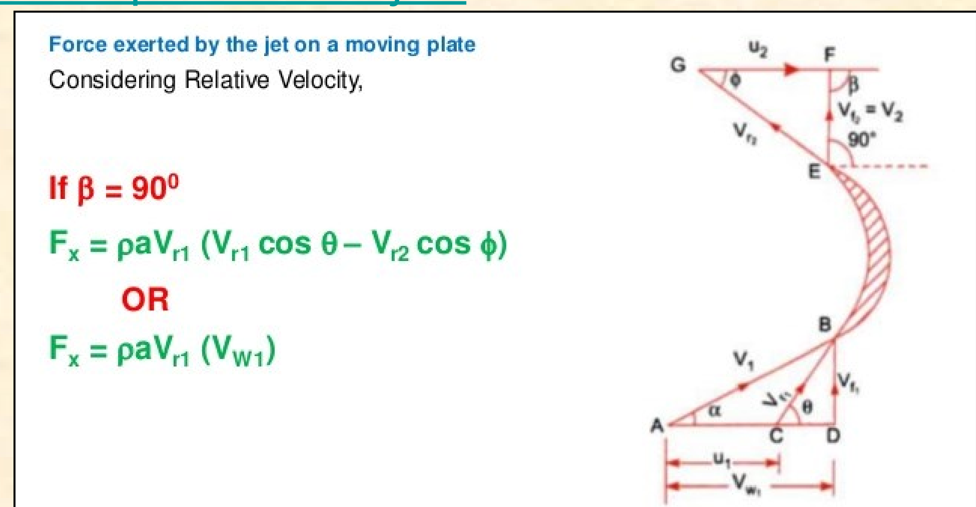
3.10 Fixing and moving vanes





- https://www.slideshare.net/RambabuPalaka/impact-of-free-jets-60682698
- https://www.slideshare.net/RambabuPalaka/impact-of-free-jets





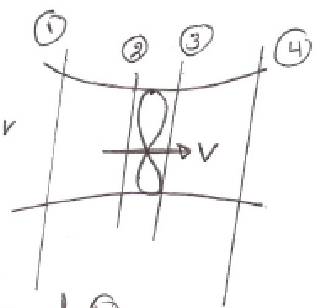
3.11 The moment theory for propeller

The action of propeller is to change the momentum of the fluid with which it is submerged and develop a thrust force that is used for propulsion.

$P_1 > P_2$, $P_3 > P_4$

$P_1 = P_4$ $\quad$ **$P_1=P_4=P_{atm}$**

$F = \rho Q (V_4 - V_1) = (P_3 - P_2) A$ --- (3-36)



$Q = AV$

$\rho V (V_4 - V_1) = P_3 - P_2$ --- (3-27)

B.E between ① and ②, ③ and ④

$$\left.\begin{aligned} P_1 + \tfrac{1}{2}\rho V_1^2 &= P_2 + \tfrac{1}{2}\rho V_2^2 \\ P_3 + \tfrac{1}{2}\rho V_3^2 &= P_4 + \tfrac{1}{2}\rho V_4^2 \end{aligned}\right\} \quad \text{--- (3-28)}$$

Since $Z_1 = Z_2 = Z_3 = Z_4$ and $P_1 = P_4$, $V_3 = V_2$

$P_3 - P_2 = \frac{1}{2}\rho (V_4^2 - V_1^2)$ --- (3-29)

from equ (3-27) and (3-29)

$\frac{1}{2}\rho (V_4^2 - V_1^2) = \rho V (V_4 - V_1)$

or

$\frac{1}{2}(V_4 - V_1)(V_4 + V_1) = V (V_4 - V_1)$

$$\boxed{\therefore V = \frac{V_4 + V_1}{2}} \quad \text{--- (3-30)}$$

سرعة المروحة
أو سرعة الهواء
عبر المروحة

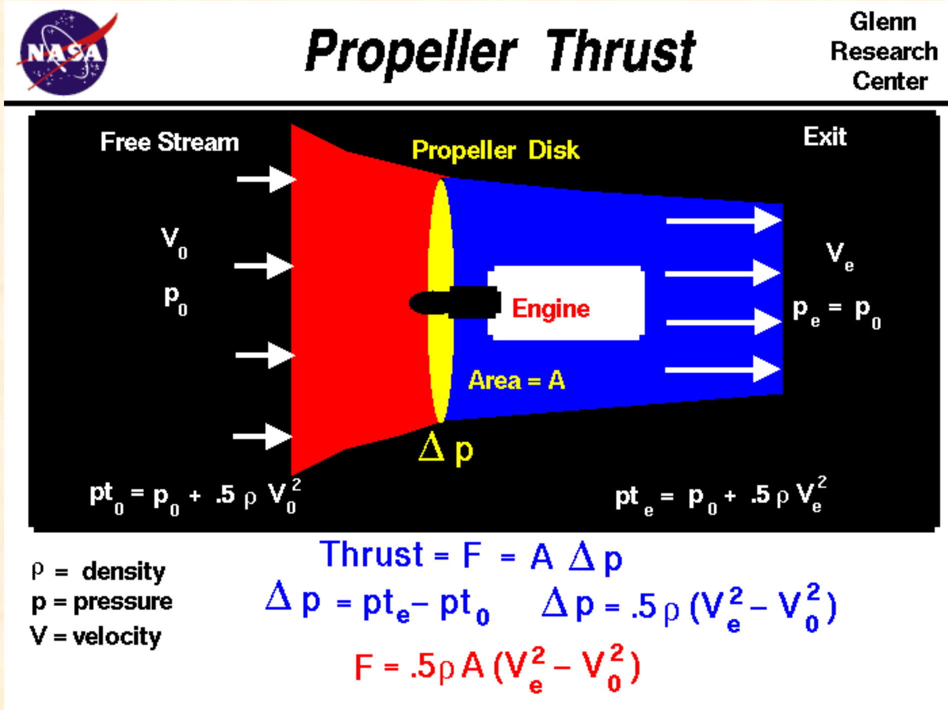
$A_2 < A_1$

$V_2 > V_1$

$P_2 < P_1$!

Bernoulli Equation: $H_1 = H_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_2$$



https://www.grc.nasa.gov/WWW/K-12/airplane/propth.html

3.11 The moment theory for propeller

(27)

$$P_{out} = \Delta K.E = \tfrac{1}{2}\rho Q (V_1^2 - V_4^2)$$

$$P_{in} = \tfrac{1}{2}\rho Q V_1^2 \quad \text{since } Q_{in} = A V_1$$

$$P_{in} = \tfrac{1}{2}\rho A V_1^3$$

$$\therefore \eta_m = \frac{P_{out}}{P_{in}} = \frac{\tfrac{1}{2}\rho Q (V_1^2 - V_4^2)}{\tfrac{1}{2}\rho A V_1^3} = \frac{Q (V_1^2 - V_4^2)}{A V_1^3}$$

$$Q = A\,V_{mean} = A * \frac{V_4 + V_1}{2}$$

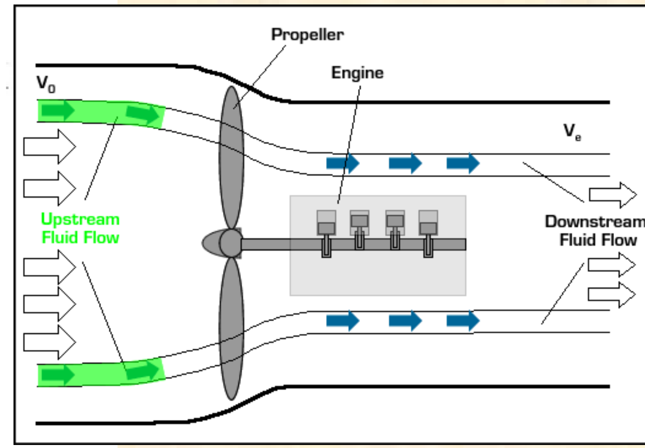
$$\text{where } \frac{Q}{A} = \frac{V_4 + V_1}{2} \Rightarrow \eta_m = \frac{(V_4 + V_1)(V_1^2 - V_4^2)}{2 V_1^3}$$

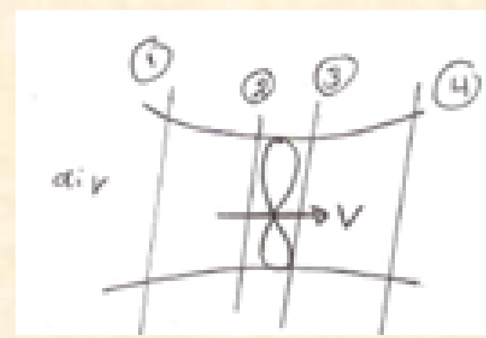
for max η_m

$$\frac{d\eta}{dV_4} = 0 = V_1^2 - 3V_4^2 - 2V_1 V_4$$

$$= (V_1 - 3V_4)(V_4 + V_1) = 0 \Rightarrow V_1 = 3V_4 \ (max), \quad V_1 = -V_4 \ (min)$$

$$\eta_m = \frac{(V_4 + 3V_4)(9V_4^2 - V_4^2)}{2(3V_4)^3} = \frac{4V_4 * 8V_4^2}{54 V_4^3} = 59.25\%$$





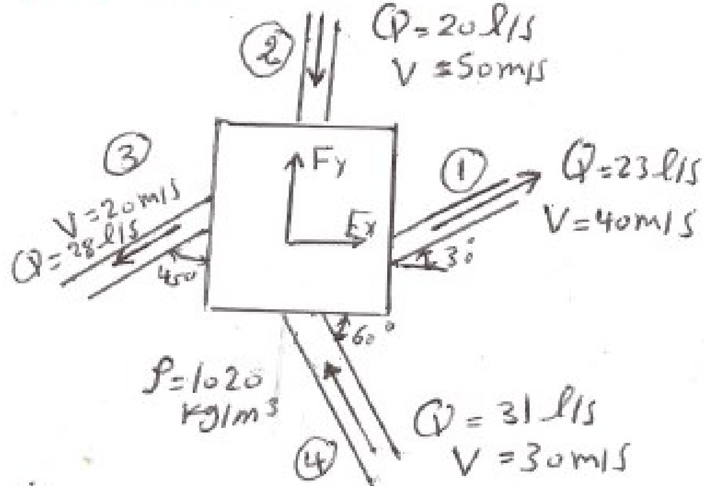
Q/ Prove the η_m=59.25 % for propeller ?

https://s2.smu.edu/propulsion/Pages/propeller.htm

Examples

Example$_1$:

what force component Fx & Fy are required to hold the black box of fig stationary all pressure are zero gauge



The momentum equation for steady one-dimensional flow is

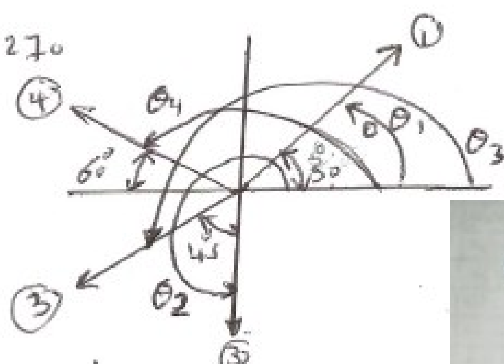
$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

Sol.$_1$:

$$\sum F_x = \sum (\rho Q V)_{out} \cos\theta - \sum (\rho Q V)_{in} \cos\theta$$

$$= 1020 \left[\left(\frac{23}{1000} * 40 * \cos 30 + \frac{28}{1000} * 20 * \cos 225 \right)_{out} - \left(\frac{20}{1000} * 50 \cos 270 + \frac{31}{1000} * 30 * \cos 120 \right)_{in} \right]$$

$$\sum F_x = 883.32 \text{ N}$$



$$\sum F_y = \sum (\rho Q V)_{out} \sin\theta - \sum (\rho Q V)_{in} \sin\theta$$

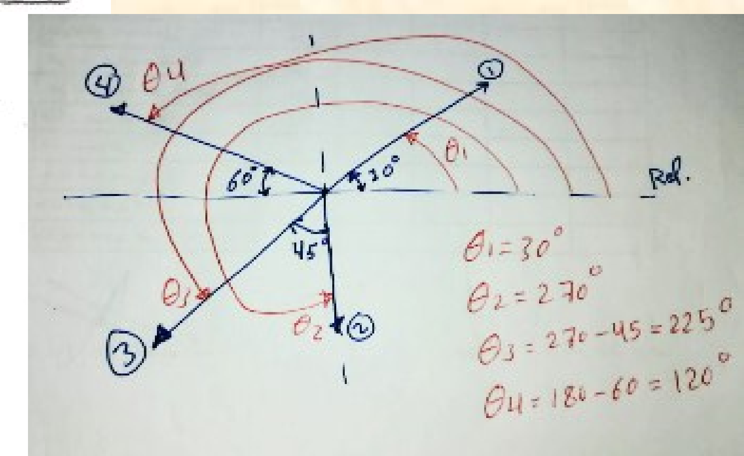
$$= 1020 \left[\left(\frac{23}{1000} * 40 * \sin 30 + \frac{28}{1000} * 20 * \sin 225 \right)_{out} - \left(\frac{20}{1000} * 50 \sin 270 + \frac{31}{1000} * 30 * \sin 120 \right)_{in} \right] \Rightarrow \sum F_y = 265.2 \text{ N}$$

$\theta_1 = 30$

$\theta_2 = 270°$

$\theta_3 = 225°$

$\theta_4 = 120°$



Example$_2$: The Force to Hold a Deflector Elbow in Place A reducing elbow is used to deflect water flow at a rate of 14 kg/s in a horizontal pipe upward 30° while accelerating it (Fig. 6–20). The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is 113 cm^2 at the inlet and 7 cm^2 at the outlet. The elevation difference between the centers of the outlet and the inlet is 30 cm. The weight of the elbow and the water in it is considered to be negligible. Determine (a) the gage pressure at the center of the inlet of the elbow and (b) the anchoring force needed to hold the elbow in place. The momentum flux correction factor (β)=1.03

Sol.$_2$: ***Properties:*** density of water is ρ_w = 1000 kg/m^3 .
Note: The continuity equation for this one-inlet, one-outlet, steady-flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$=14 kg/s. Noting that $\dot{m} = \rho A V$, the inlet and outlet velocities of water are

(a):

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{14 \text{ kg/s}}{(1000 \text{ kg/m}^3)(0.0113 \text{ m}^2)} = 1.24 \text{ m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{14 \text{ kg/s}}{(1000 \text{ kg/m}^3)(7 \times 10^{-4} \text{ m}^2)} = 20.0 \text{ m/s}$$

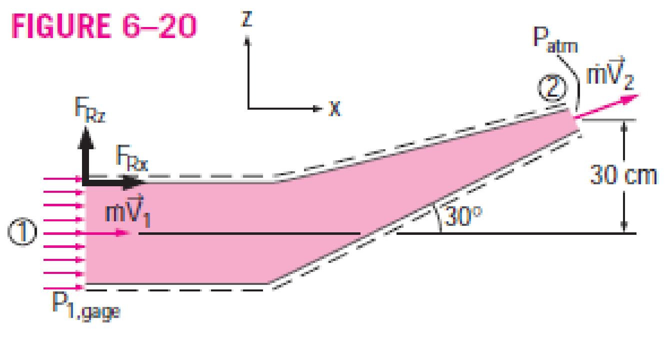
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$P_1 - P_2 = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2 - z_1\right)$$

$$P_1 - P_{atm} = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \times \left(\frac{(20 \text{ m/s})^2 - (1.24 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0.3 - 0\right)\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)$$

$$P_{1,\text{ gage}} = 202.2 \text{ kN/m}^2 = 202.2 \text{ kPa} \quad \text{(gage)}$$

FIGURE 6–20

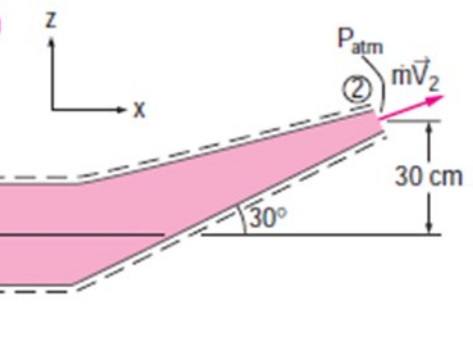


Examples

Sol.$_2$: (b)

(b) The momentum equation for steady one-dimensional flow is

$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$



$$F_{Rx} + P_{1,\,gage}A_1 = \beta \dot{m} V_2 \cos\theta - \beta \dot{m} V_1$$

$$F_{Rz} = \beta \dot{m} V_2 \sin\theta$$

Solving for F_{Rx} and F_{Rz}, and substituting the given values,

$$F_{Rx} = \beta \dot{m}(V_2 \cos\theta - V_1) - P_{1,\,gage}A_1$$

$$= 1.03(14\ \text{kg/s})[(20 \cos 30° - 1.24)\ \text{m/s}]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$- (202{,}200\ \text{N/m}^2)(0.0113\ \text{m}^2)$$

$$= 232 - 2285 = \mathbf{-2053\ N}$$

$$F_{Rz} = \beta \dot{m} V_2 \sin\theta = (1.03)(14\ \text{kg/s})(20 \sin 30°\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{144\ N}$$

Examples

Example$_3$: A 90° elbow is used to direct water flow at a rate of **25 kg/s** in a horizontal pipe upward. The diameter of the entire elbow is 10 cm. The elbow discharges water into the atmosphere, and thus the pressure at the exit is the local atmospheric pressure. The elevation difference between the centers of the exit and the inlet of the elbow is 35 cm. The weight of the elbow and the water in it is considered to be negligible. Determine (a) the gage pressure at the center of the inlet of the elbow and (b) the anchoring force needed to hold the elbow in place. Take the momentum-flux correction factor (β) to be 1.03. ; ***Properties:*** density of water is ρ_w = 1000 kg/m^3 .

Sol.$_3$:

Analysis (*a*) We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction) and the vertical coordinate by *z*. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30$ kg/s. Noting that $\dot{m} = \rho A V$, the mean inlet and outlet velocities of water are

$$V_1 = V_2 = V = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{25\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.1\,\text{m})^2/4]} = 3.18\ \text{m/s}$$

Noting that $V_1 = V_2$ and $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \;\rightarrow\; P_1 - P_2 = \rho g(z_2 - z_1) \;\rightarrow\; P_{1,\,gage} = \rho g(z_2 - z_1)$$

Substituting,

$$P_{1,\,gage} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.35\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 3.434\ \text{kN/m}^2 = 3.434\ \text{kPa} \cong \mathbf{3.43\ kPa}$$

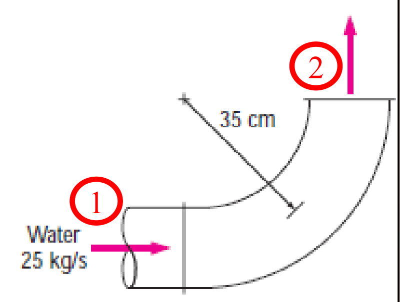


FIGURE P6–22

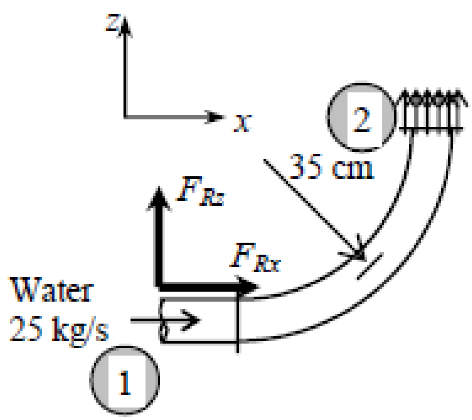
Examples

(*b*) The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$. We let the *x*- and *z*-components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the *x* and *y* axes become

$$F_{Rx} + P_{1,\text{gage}} A_1 = 0 - \beta \dot{m}(+V_1) = -\beta \dot{m} V$$

$$F_{Rz} = \beta \dot{m}(+V_2) = \beta \dot{m} V$$

Sol.$_3$:



Solving for F_{Rx} and F_{Rz}, and substituting the given values,

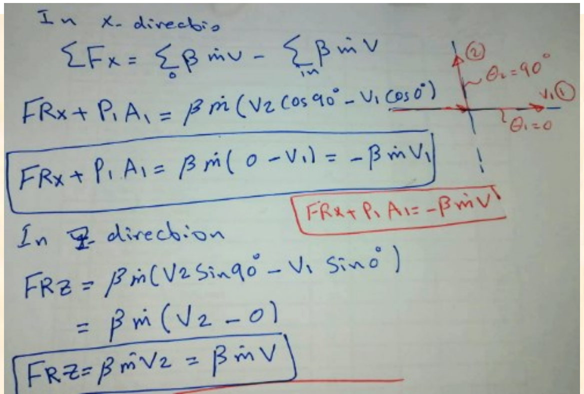
$$F_{Rx} = -\beta \dot{m} V - P_{1,\text{gage}} A_1$$

$$= -1.03(25\text{ kg/s})(3.18\text{ m/s})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) - (3434\text{ N/m}^2)[\pi(0.1\text{ m})^2/4]$$

$$= -109\text{ N}$$

$$F_{Ry} = \beta \dot{m} V = 1.03(25\text{ kg/s})(3.18\text{ m/s})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 81.9\text{ N}$$

and $F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-109)^2 + 81.9^2} = \mathbf{136\ N}$, $\theta = \tan^{-1}\dfrac{F_{Ry}}{F_{Rx}} = \tan^{-1}\dfrac{81.9}{-109} = -37° = \mathbf{143°}$



Examples

Example$_4$: A reducing elbow is used to deflect water flow at a rate of **30 kg/s** in a horizontal pipe upward by an angle **θ = 45°** from the flow direction while accelerating it. The elbow discharges water into the atmosphere. The cross sectional area of the elbow is 150 cm^2 at the inlet and 25 cm^2 at the exit. The elevation difference between the centers of the exit and the inlet is 40 cm. The mass of the elbow and the water in it is 50 kg. Determine the anchoring force needed to hold the elbow in place. Take the momentum-flux correction factor to be 1.03.

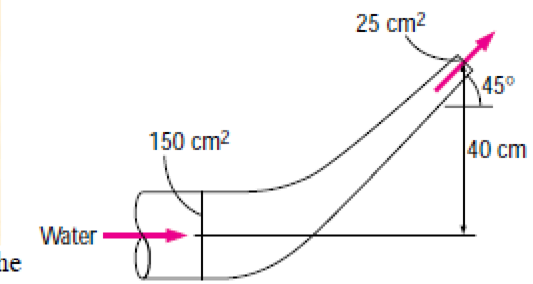


FIGURE P6–25

Sol.$_4$:

Solution A reducing elbow deflects water upwards and discharges it to the atmosphere at a specified rate. The anchoring force needed to hold the elbow in place is to be determined.

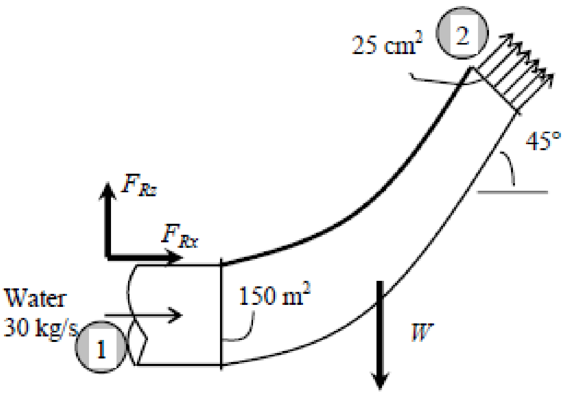
Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is considered. **3** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **4** The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.03$.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The weight of the elbow and the water in it is

$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2) = 490.5\text{ N} = 0.4905\text{ kN}$$

We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction) and the vertical coordinate by z. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30$ kg/s. Noting that $\dot{m} = \rho A V$, the inlet and outlet velocities of water are



$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0150\text{ m}^2)} = 2.0\text{ m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0025\text{ m}^2)} = 12\text{ m/s}$$

Examples

The momentum equation for steady one-dimensional flow is

$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

Sol.$_4$:

Taking the center of the inlet cross section as the reference level ($z_1 = 0$) and noting that $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \;\rightarrow\; P_1 - P_2 = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2 - z_1\right) \;\rightarrow\; P_{1,\,gage} = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2\right)$$

Substituting,

$$P_{1,\,gage} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left(\frac{(12\ \text{m/s})^2 - (2\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 0.4\right)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 73.9\ \text{kN/m}^2 = 73.9\ \text{kPa}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$. We let the x- and z- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the x and z axes become

$$F_{Rx} + P_{1,gage} A_1 = \beta \dot{m} V_2 \cos\theta - \beta \dot{m} V_1 \quad \text{and} \quad F_{Rz} - W = \beta \dot{m} V_2 \sin\theta$$

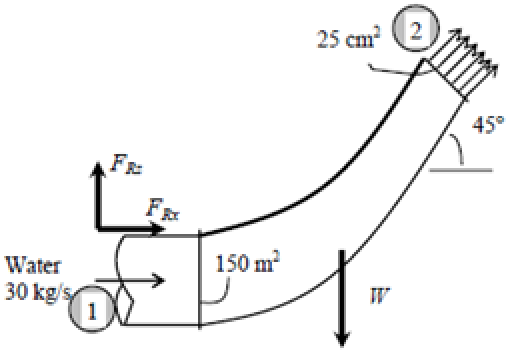
Solving for F_{Rx} and F_{Rz}, and substituting the given values,

$$F_{Rx} = \beta \dot{m}\left(V_2 \cos\theta - V\right)_1 - P_{1,\,gage} A_1 = 1.03(30\ \text{kg/s})[(12\cos 45°\text{-}2)\ \text{m/s}]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) - \left(73.9\ \text{kN/m}^2\right)\left(0.0150\ \text{m}^2\right)$$

$$= -0.908\ \text{kN}$$

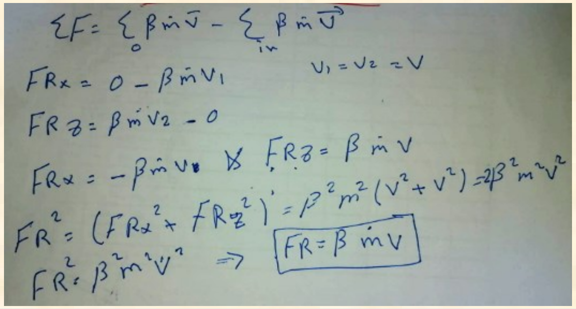
$$F_{Rz} = \beta \dot{m} V_2 \sin\theta + W = 1.03(30\ \text{kg/s})(12\sin 45°\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) + 0.4905\ \text{kN} = 0.753\ \text{kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2} = \sqrt{(-0.908)^2 + (0.753)^2} = \mathbf{1.18\ kN}, \quad \theta = \tan^{-1}\frac{F_{Rz}}{F_{Rx}} = \tan^{-1}\frac{0.753}{-0.908} = \mathbf{-39.7}$$



Examples

Example$_5$: Water Jet Striking a Stationary Plate; Water is accelerated by a nozzle to an average speed of 20 m/s, and strikes a stationary vertical plate at a rate of 10 kg/s with a normal velocity of 20 m/s (Fig. 6–22). After the strike, the water stream splatters off in all directions in the plane of the plate, and β=1. Determine the force needed to prevent the plate from moving horizontally due to the water stream.



Sol.$_5$:

Analysis We draw the control volume for this problem such that it contains the entire plate and cuts through the water jet and the support bar normally. The momentum equation for steady one-dimensional flow is given as

$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

Writing it for this problem along the *x*-direction (without forgetting the negative sign for forces and velocities in the negative *x*-direction) and noting that $V_{1,x} = V_1$ and $V_{2,x} = 0$ gives

$$-F_R = 0 - \beta \dot{m} \vec{V}_1$$

Substituting the given values,

$$F_R = \beta \dot{m} \vec{V}_1 = (1)(10\ kg/s)(20\ m/s)\left(\frac{1\ N}{1\ kg \cdot m/s^2}\right) = 200\ N$$

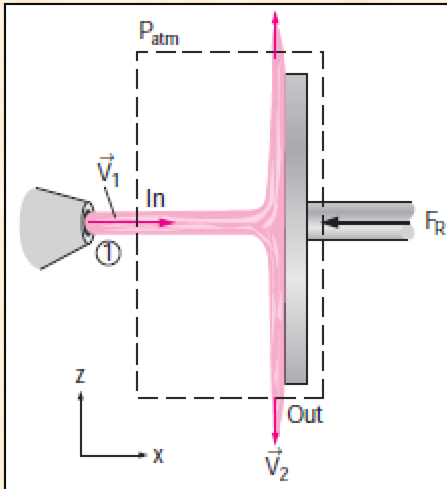
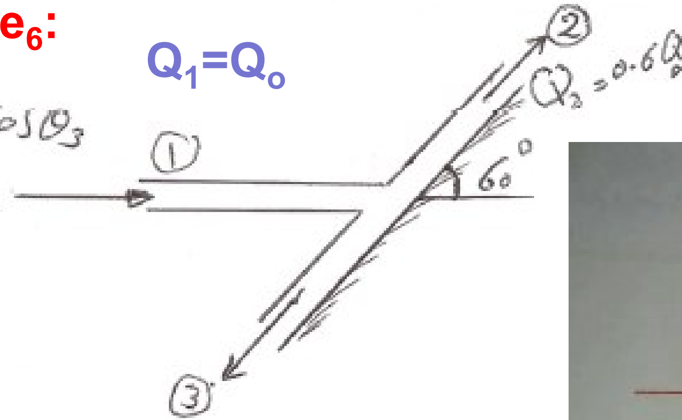


FIGURE 6–22

Examples

Calculate the force components Fx, Fy needed to hold the stationary Vane of Fig $Q_o = 80\ l/s$, $\rho = 1000\ kg/m^3$, $V_o = 120\ m/s$

Example$_6$:



$Q_1 = Q_o$

$Q_3 = 0.4\ Q_o$

The momentum equation for steady one-dimensional flow is

$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

Sol.$_6$:

$\Sigma F_x = \rho Q_2 V_2 \cos\theta_2 + \rho Q_3 V_3 \cos\theta_3 - \rho Q_1 V_1 \cos\theta_1$

$V_1 = V_2 = V_3 = V$ (open sys)

$P_1 = P_2 = P_3 = atm$

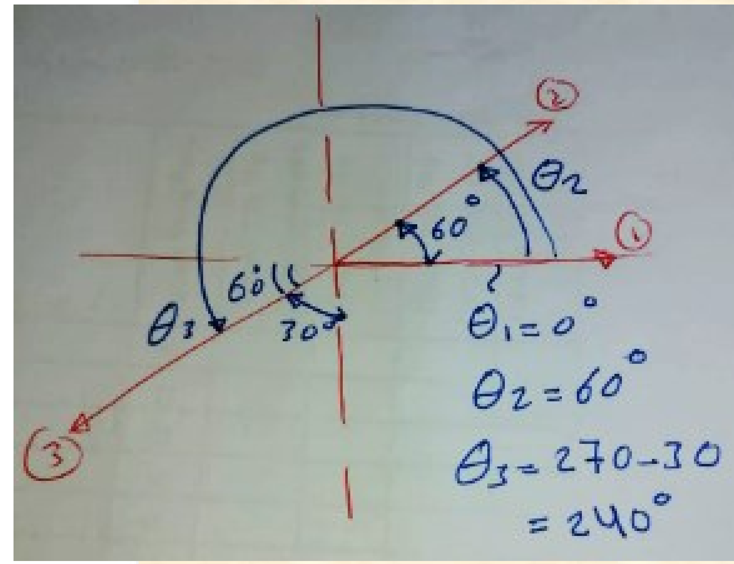
$Q_2 = 0.6 Q_o$

$Q_3 = 0.4 Q_o$

$\Sigma F_x = \rho V Q_o [0.6 \cos\theta_2 + 0.4 \cos\theta_3 - \cos\theta_1]$

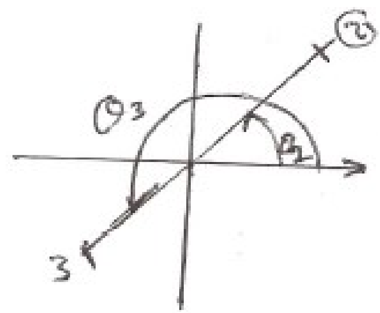
$V = 120\ m/s$, $Q_o = 0.08\ m^3/s$

$\theta_1 = 0$, $\theta_2 = 60°$, $\theta_3 = 240°$



$\Sigma F_x = -8640\ N \rightarrow$

$\therefore F_x = 8640\ N \leftarrow$



$F_y = \rho V Q [0.6 \sin\theta_2 + 0.4 \sin\theta_3 - \sin\theta_1]$

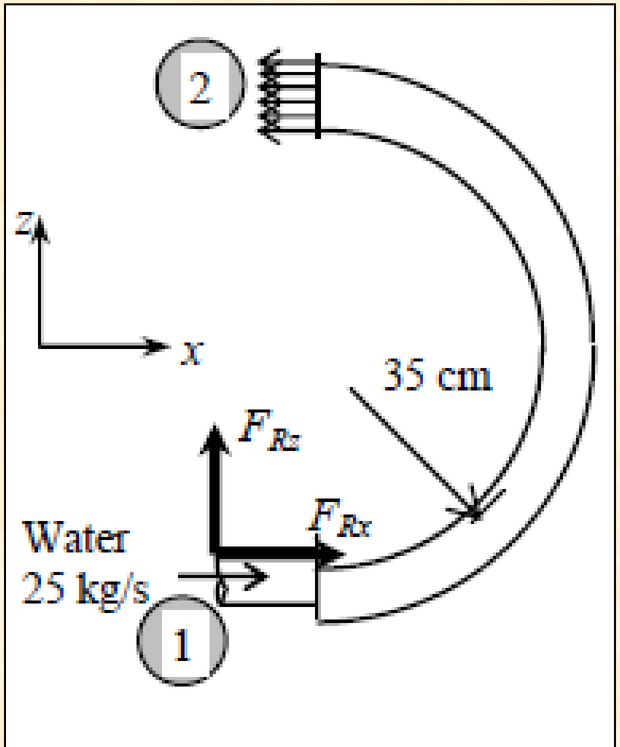
$= 1662.7\ N \uparrow$

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2}$$

$$\theta = \tan^{-1} \frac{F_{Rz}}{F_{Rx}}$$

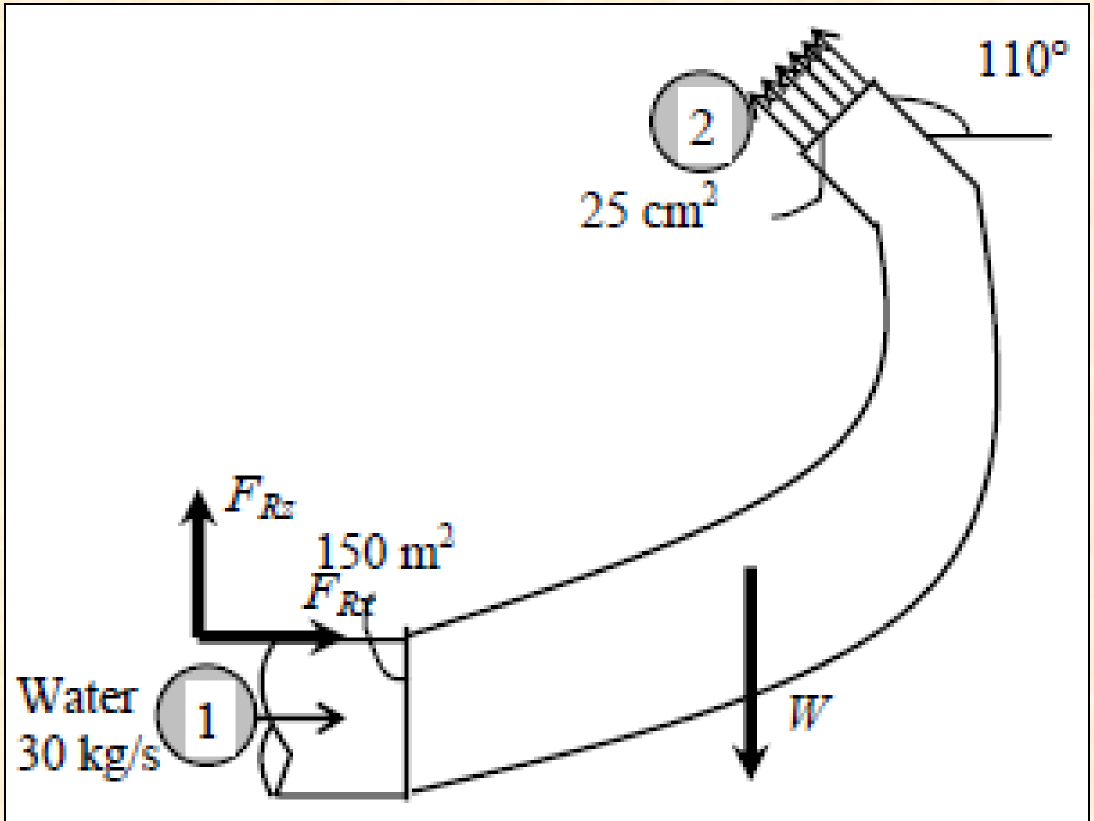
Homeworks:

Hw$_1$: A 90° elbow is used to direct water flow at a rate of **25 kg/s** in a horizontal pipe upward. The diameter of the entire elbow is 10 cm. The elbow discharges water into the atmosphere, and thus the pressure at the exit is the local atmospheric pressure. The elevation difference between the centers of the exit and the inlet of the elbow is 35 cm. The weight of the elbow and the water in it is considered to be negligible. Determine (a) the gage pressure at the center of the inlet of the elbow and (b) the anchoring force needed to hold the elbow in place. for the case of another (identical) elbow being attached to the existing elbow so that the fluid makes a U-turn. Take the momentum-flux correction factor to be **1.03**. ; ***Properties:*** density of water is ρ_w = 1000 kg/m^3 . Answers: (a) 6.87 kPa, (b) 218 N



Homeworks:

Hw$_2$: A reducing elbow is used to deflect water flow at a rate of 30 kg/s in a horizontal pipe upward by an angle $\theta = 110°$ from the flow direction while accelerating it. The elbow discharges water into the atmosphere. The cross sectional area of the elbow is 150 cm^2 at the inlet and 25 cm^2 at the exit. The elevation difference between the centers of the exit and the inlet is 40 cm. The mass of the elbow and the water in it is 50 kg. Determine the anchoring force needed to hold the elbow in place. Take the momentum-flux correction factor to be 1.03.



Homeworks:

Hw$_3$: A 100-ft^3/s water jet is moving in the positive x-direction at 20 ft/s. The stream hits a stationary splitter, such that half of the flow is diverted upward at 45° and the other half is directed downward, and both streams have a final speed of 20 ft/s. Disregarding gravitational effects, determine the x- and z-components of the force required to hold the splitter in place against the water force. Note: you have to solve the question in SI units.

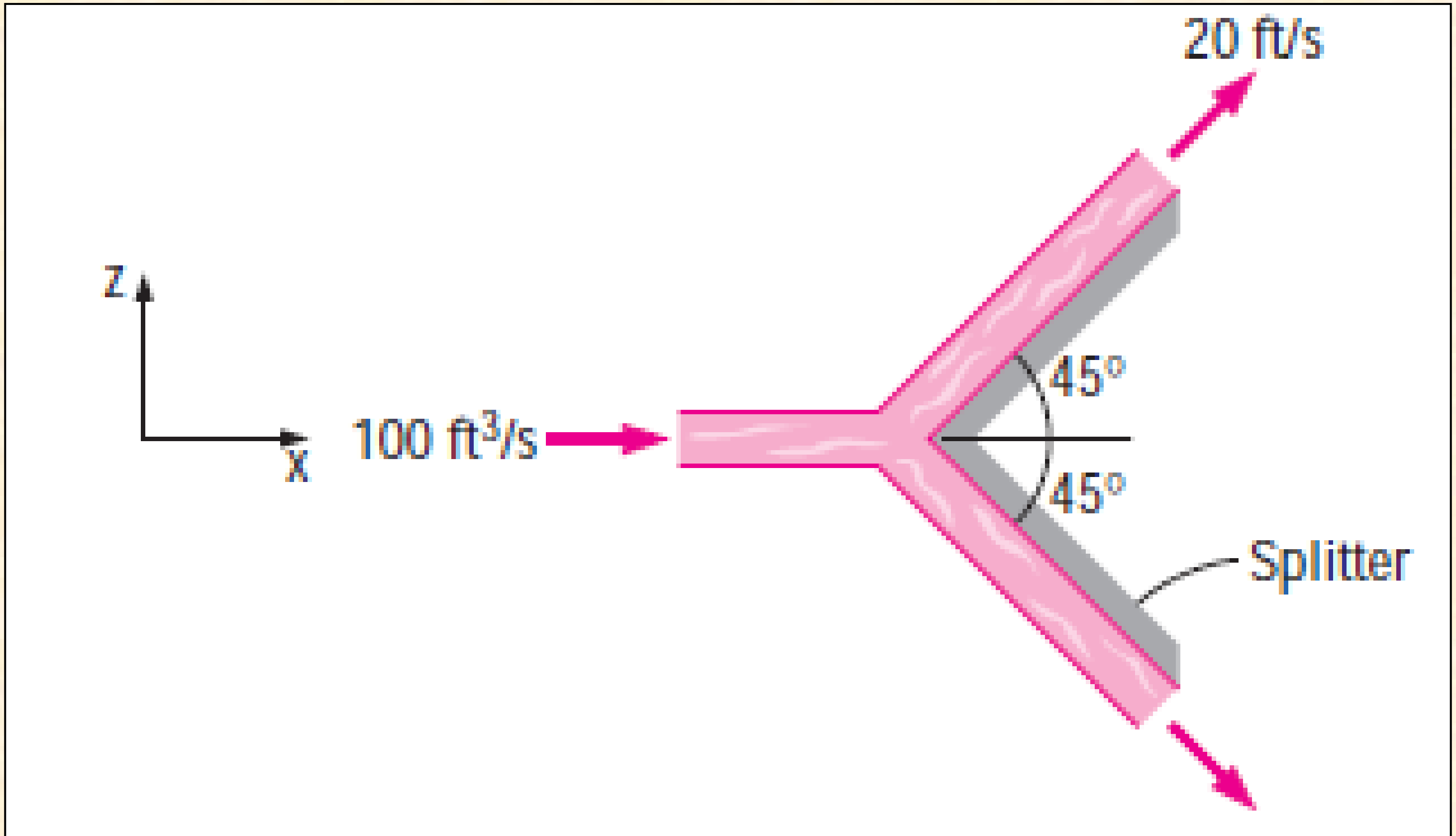


FIGURE P6–29E

Exams and Grading Policy:

❖ The distribution of Fluid mechanics degree for the students in course-1 as following the table:

Quizzes (2) 10%		Project (1) 6%		Assignments (H.Ws) 9%		Midterm exam (1) 10%	Laboratory (3) 15%		Final
(1)	(2)	Report Structures	Report Discussion	Online (5)	Onsite (2)	-	Report Structures	Report Discussion	
5 %	**5 %**	4 %	2 %	**5 %**	4 %	10 %	9 %	6 %	**50 %**

- Note: Solve all three Homeworks and sending me the answering next week on Thursday 13 March 2025.

- I hope everything is clear for all students

- Good luck